There can be three possible cases. Either, the final temperature of gas will be less than T_0, or equal to T_0, or greater than T_0. It all depends on the initial KE possessed by container. We also use the fact that temp of gas depends on the randomised molecular motion from CM frame

of container, similar to the IRODOV question.

(i) Let final temp be T_1, which is less than T_0. Then,

$$\frac{1}{2} m v^2 = \left(\frac{m}{M}\right)\left(\frac{3R}{2}\right)(T_f - T_1)$$

$$T_f = \frac{Mv^2}{3R} + T_1.$$ We also have to ensure that this is less than T_0. Hence,

$$\frac{Mv^2}{3R} + T_1 < T_0$$

$$v \leq \sqrt{\frac{3R(T_0 - T_1)}{M}}.$$

(ii) Final temp is T_0. For this, the change of internal energy must be $\frac{5nR}{2}T_0 - \frac{3nR}{2}T_1$.

$$\frac{1}{2}mv^2 = \frac{5m}{M}\frac{R}{2}T_0 - \frac{3m}{M}\frac{R}{2}T_1$$

$$v = \sqrt{\frac{5RT_0 - 3RT_1}{M}}$$. Now here comes the interesting part. For all $T \neq T_0$, there will be a one-one relationship between initial 'v' & final temp. However,

$T = T_0$ is the boundary condition. Here at $T = T_0$, DOF will unfold. This means may be not all molecules will have DOF = '5'. That will depend on initial velocity. We can say that with

$$v = \sqrt{\frac{R}{M}(5T_0 - 3T_1)}$$, 'ALL' gas molecules will have their rotational 'DOF' unfrozen. However for

$\sqrt{\frac{3R}{M}(T_0 - T_1)} < v < \sqrt{\frac{R}{M}(5T_0 - 3T_1)}$,

The gas will have temp 'T_0', with mixture of molecules having DOF '3' & DOF '5'. Beyond this velocity, the final temp will be $\frac{3T_1}{5} + \frac{Mv^2}{5R}$, as can be easily obtained.